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Multifractal growth in periphyton communities. –

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Appendix 1

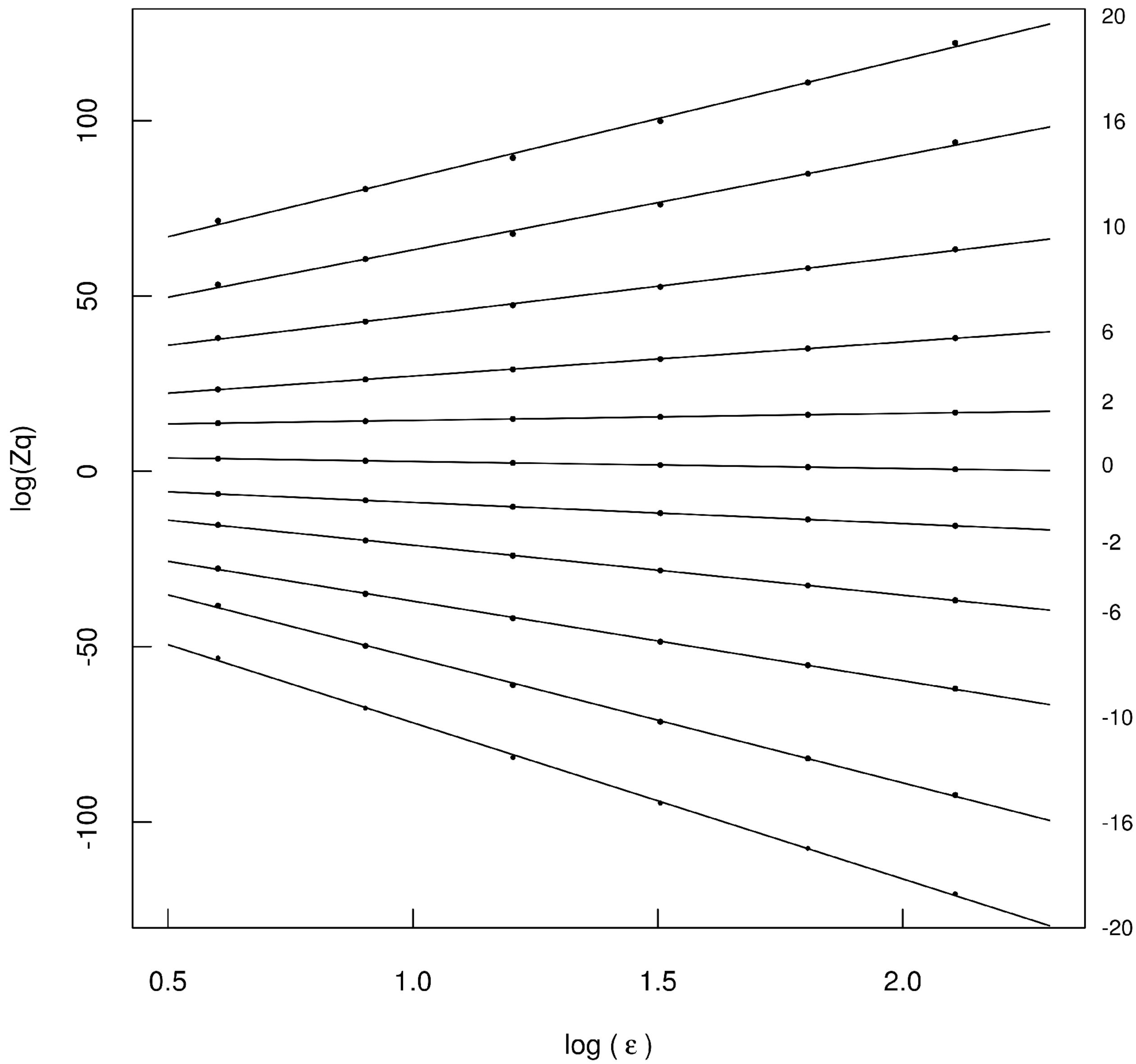

Figure A1. Example of a typical graph for determination of the generalized dimension. D_q are obtained from the slopes of the linear regression for each q for a range of -20 to 20. The numbers on the right correspond to q, each ordinate is slightly shifted from the origin for better visualization.

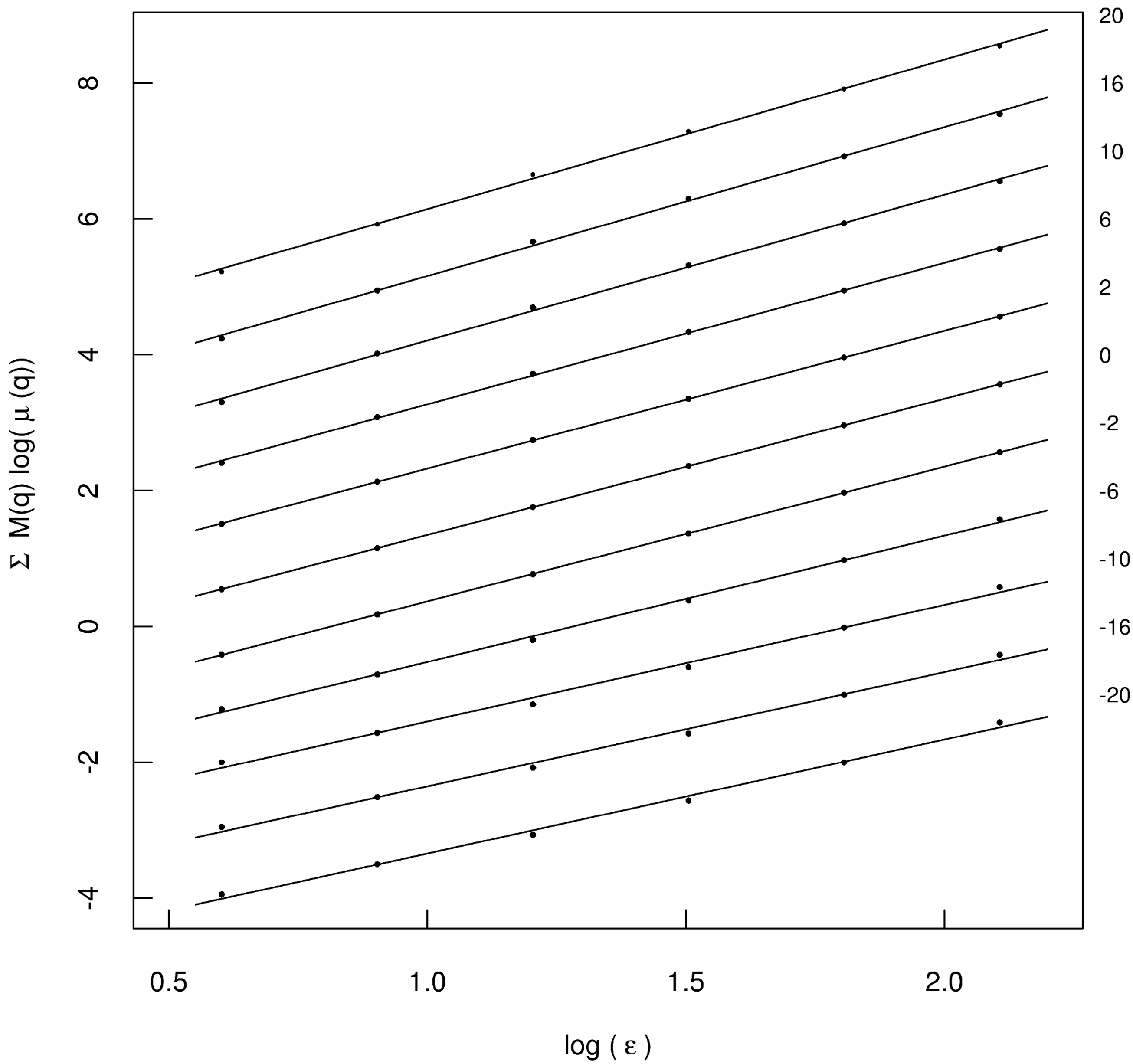

Figure A2. Example of a typical graph to determine the exponent of singularity α. The slopes of the linear regressions are used to calculate α, the values to the right correspond to the q used. Each ordinate is slightly shifted from the origin for better visualization.

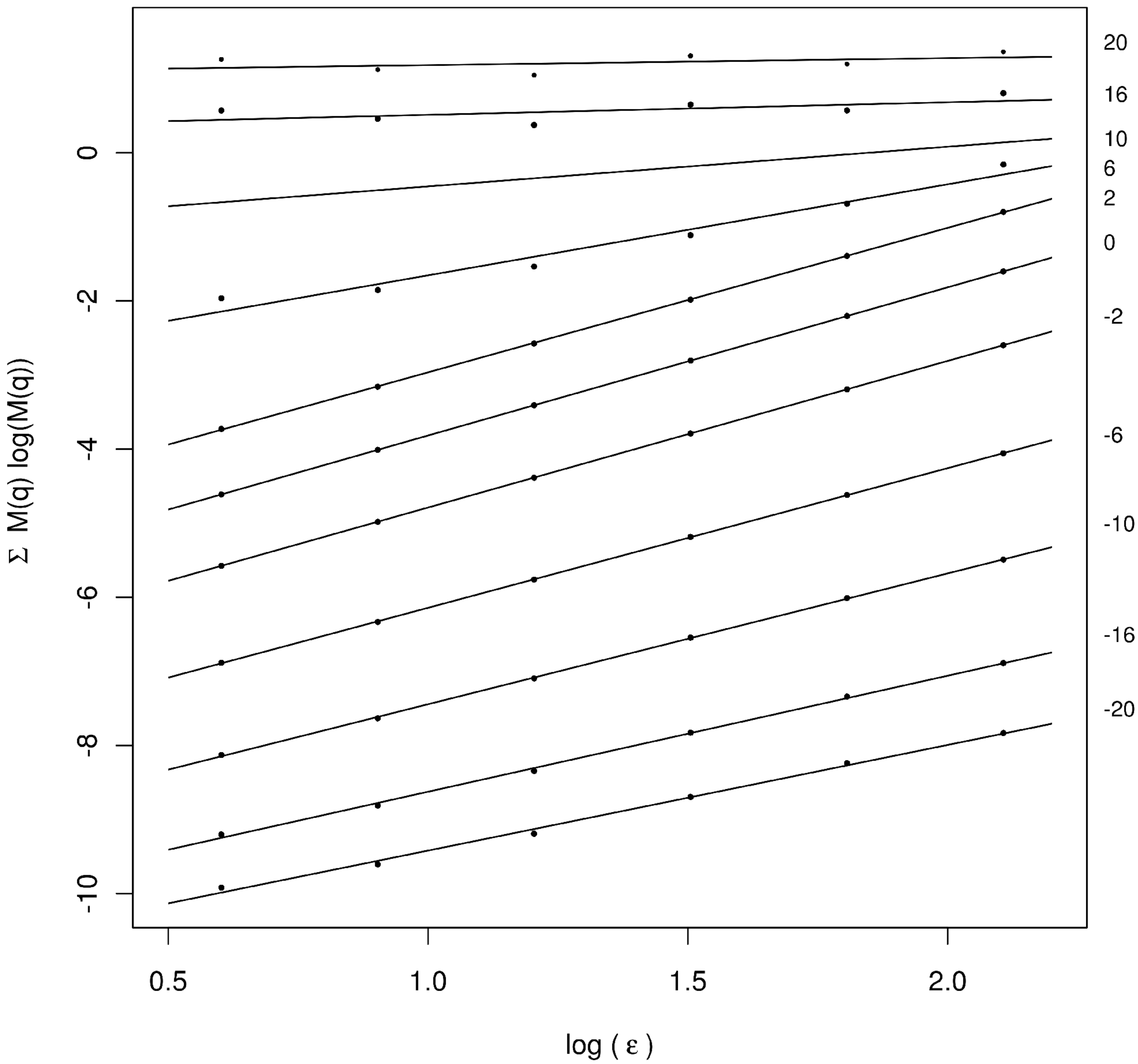

Figure A3. Example of a typical graph for determination of the fractal dimension *f(α)*. The slopes of the linear regressions are used to calculate *f(α)*, the values to the right correspond to the *q* used. This graph corresponds to the last weeks of community development where we can observe good fits for smaller *q* and some deviations from linear behavior for negative values of *q*.

Appendix 2

Estimation of the spectrum of singularities

We use the canonical method to estimate the spectrum of singularities, avoiding the use of a Legendre transformation of D_q (Chhabra and Jensen 1989). The method is similar to that used for the generalized dimensions. The biomass distribution is covered with a grid that divides the image into *N(ε)* equal squares of side *ε*. Then the biomass $\mu_i(\varepsilon)$ in each of them is used to calculate a standardized biomass:

$$M_i(q,\varepsilon)=\frac{\left[u_i(\varepsilon)\right]^q}{\sum_{j=1}^{N(\varepsilon)}\left[u_j(\varepsilon)\right]^q}$$

similarly to the generalized dimensions, the *q* parameter acts like a microscope for exploring different regions of the distribution of biomass. For $q>1$, *M(q)* amplifies the regions with more singularities, that have a lower singularity exponent. While for $q<1$ it accentuates the more uniform regions, with higher singularity exponent, and for $q = 1$, it replicates the original biomass distribution.

Then the fractal dimension of *M(q)* is calculated as:

$$f(q)=\lim_{\varepsilon\to 0}\frac{\sum_{i=1}^{N(\varepsilon)}M_i(q,\varepsilon)\log(M_i(q,\varepsilon))}{\log(\varepsilon)}$$

The average value of the exponent of singularity with respect to *M(q)* can also be calculated by evaluating:

$$\alpha(q)=\lim_{\varepsilon\to 0}\frac{\sum_{i=1}^{N(\varepsilon)}M_i(q,\varepsilon)\log(u_i(\varepsilon))}{\log(\varepsilon)}$$

The values of α and *f(α)* are estimated as the slope of the numerator versus *log(ε)*, this relationship should be linear for the biomass distribution to be multifractal. Chhabra & Jensen (1989) applied this method to theoretical multifractal measures and demonstrated that the choice of the size of the boxes *ε* for which *α* and *f(α)* is evaluated influences the estimation and can produce errors if the choice is not adequate. When the process that generates multifractal distribution is unknown there is no way to determine what would be the correct size. But despite these errors the method can reproduce most of the spectrum with accuracy, especially at the smaller absolute *q* values. For greater *q* values, oscillations in the logarithmic graphs can cause errors in the estimation of slopes.

References

Chhabra, A. B. and Jensen, R. V. 1989. Direct determination of the f(alfa) singularity espectrum. - Physical Review Letters 62: 1327-1330.